On the Seismic Wave Propagation Simulation using Boundary Element Method

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Domain methods have reached at practical application

Finite Element Method
Finite Difference Methods

Boundary methods stay in the stage of research, yet.

Boundary Element Method
Boundary Integral Equation Methods

This is mainly due to the heavy requirement of computer resource and long calculation time.

The boundary methods are still attracting researchers,
because of the potential benefits.

Benefit is expected,

if the boundary methods can do something that the domain methods can not do, ...

if the requirement for computer resource and calculation time get much reduced , ...

Example of something that the domain methods can not do

the reflection and transmission operator
in the space domain

Extension of wave propagation theory for horizontally layered media.

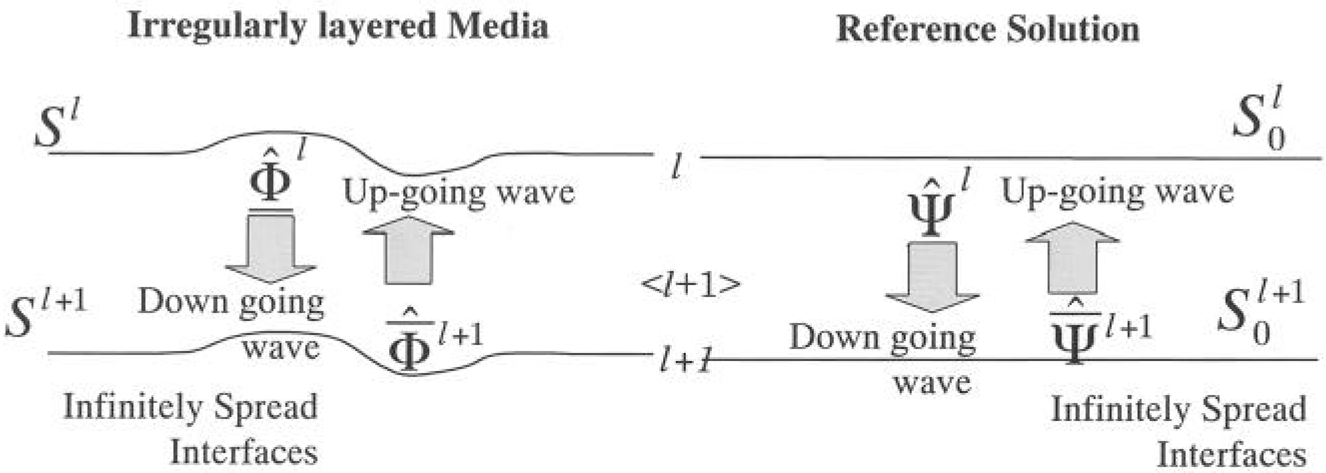
Reflection and Transmission Matrix Method (Kennett(1983))
to the irregularly stratified media

These allow to separate wave types in terms of reflection and transmission as well as the ray theory can do.

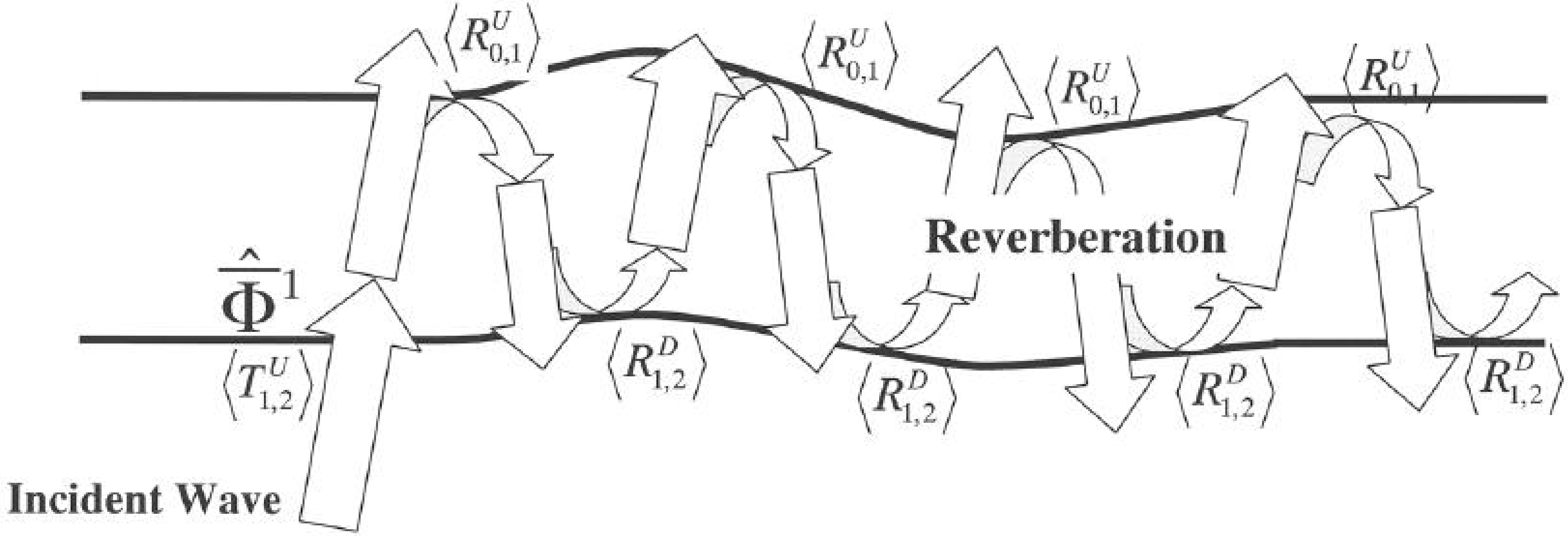
Geophys. Jour. Int., Vol. 148, No.1, 88-102, 2002

Idea:

The Indirect Boundary Element Method (I-BEM) is one of the ways to disassemble the wave field in up and down going waves in layers sandwiched between irregular interfaces. Green's function matrices of I-BEM play the role of the wave function of Reflection and Transmission Matrix Method for horizontally layered media, and the imaginary forces distributed along both faces of interfaces that of the coefficients vectors.



Multiple Reflection in the surface layer



Numerical Example: homogeneous Semi-cylindrical 2D basin

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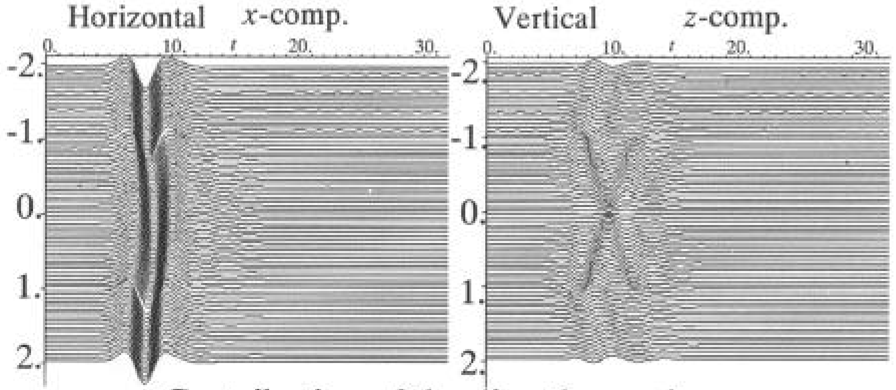
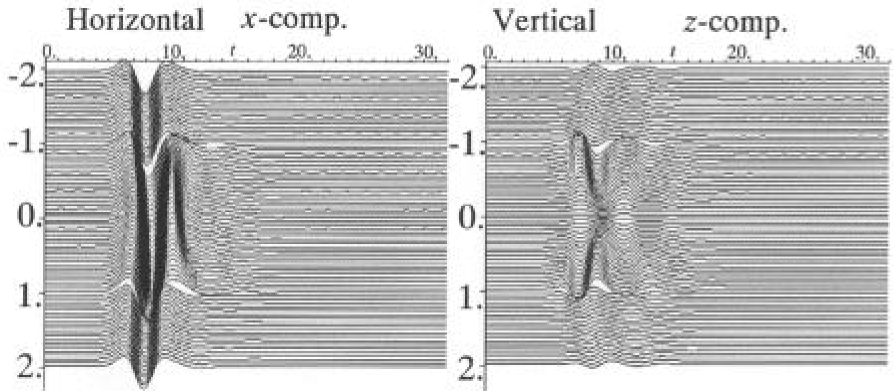
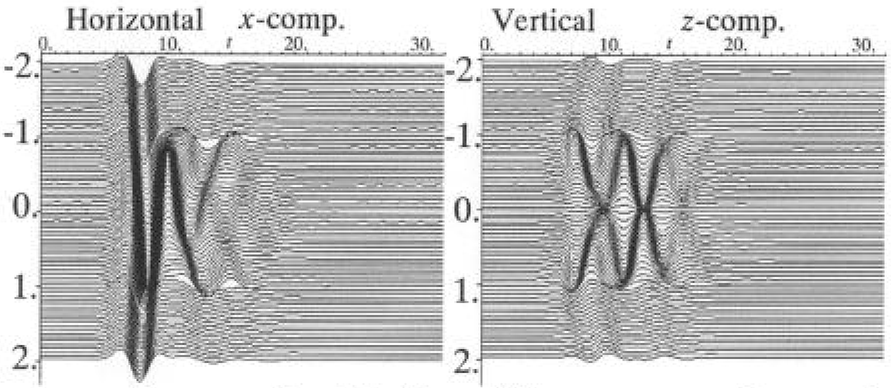


Contribution of the directly coming wave

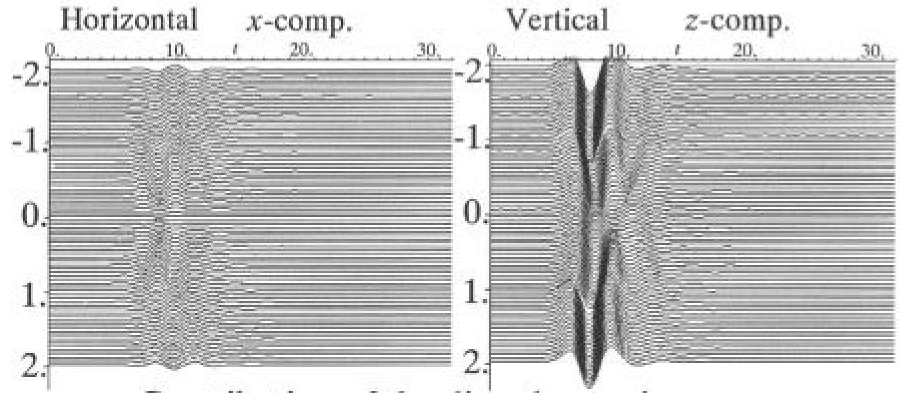


Above + Contribution of the surface reflection

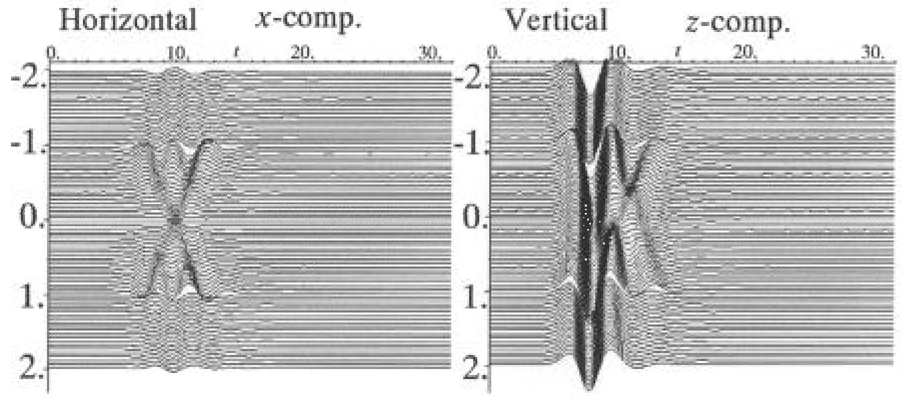


Above + Contribution of the wave once bounced

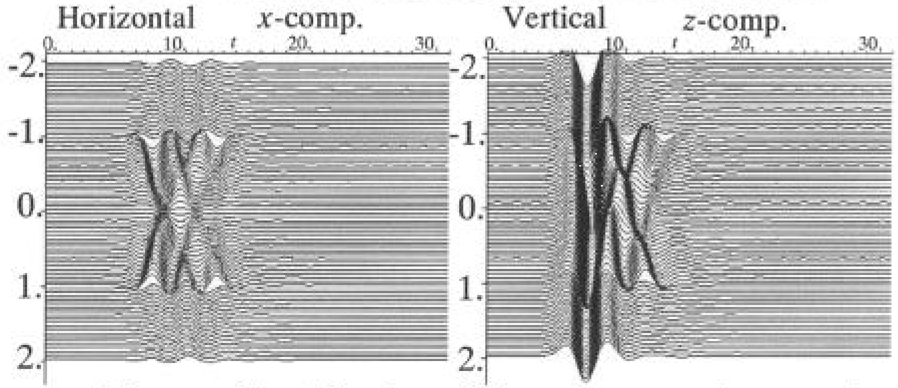
Vertical Incidence of SV-Wave



Contribution of the directly coming wave

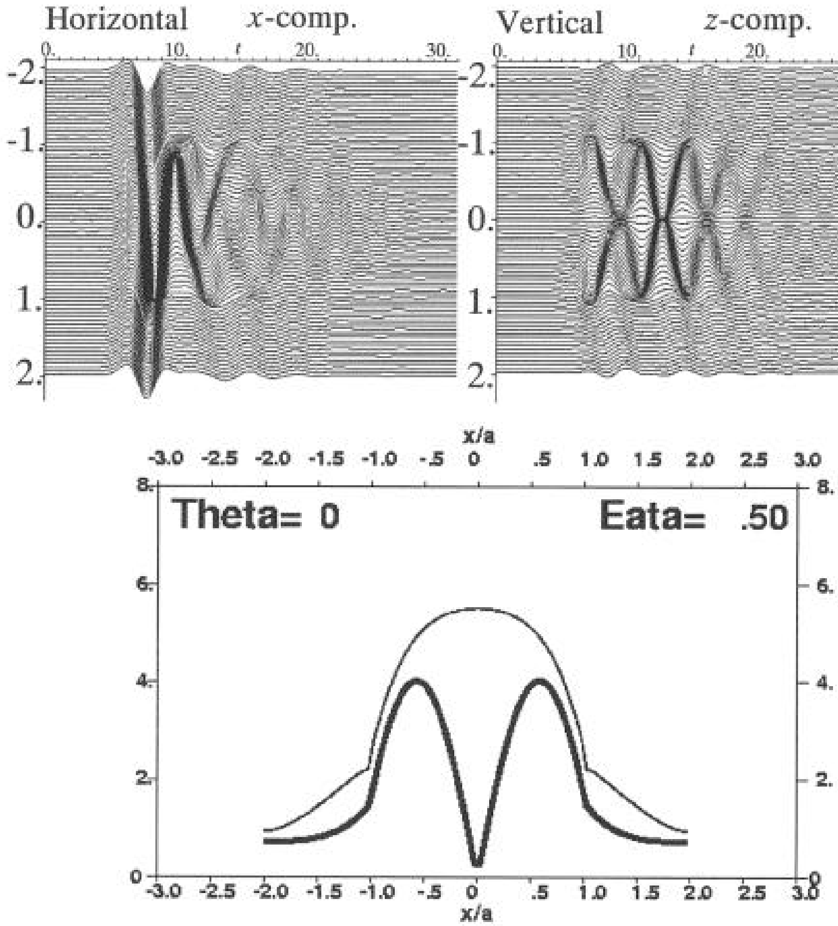


Above + Contribution of the surface reflection

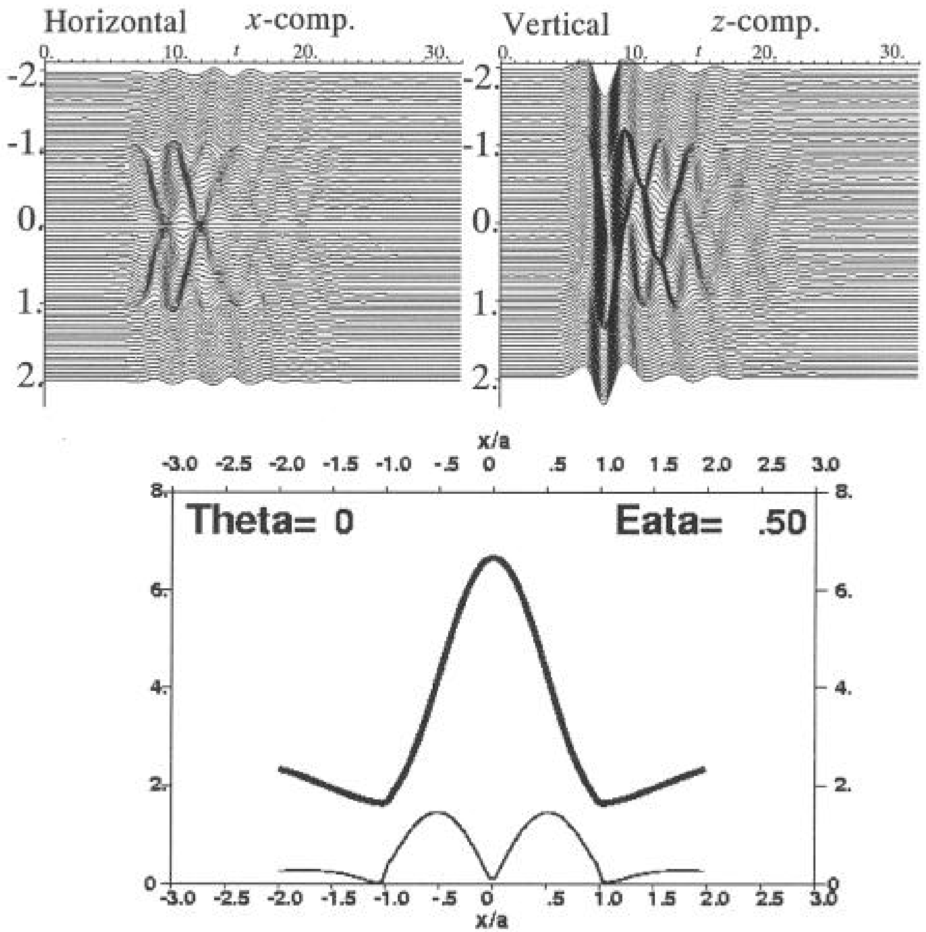


Above + Contribution of the wave once bounced

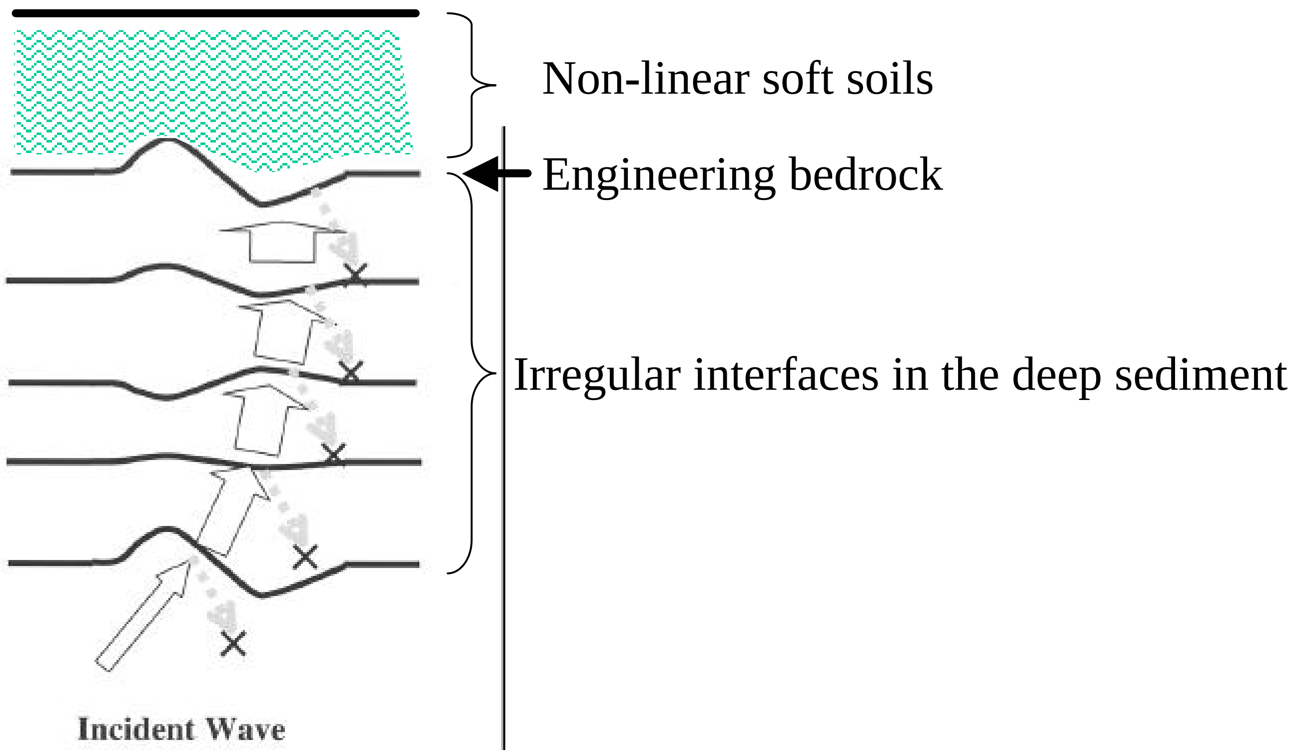
Vertical Incidence of P-Wave



Vertical Incidence of SV Wave



Vertical Incidence of P Wave


Non-linear soft soils
Engineering bedrock
Irregular interfaces in the deep sediment
Incident Wave

Effort to reduce the requirement for computer resource and calculation time: → Sparse Matrix Approximation

$$Ac = -g \qquad\qquad A^{(0)}c^{(0)} = -g$$

Extract the matrix elements of which absolute value is considerable. This approximation reduces the required main memory and CPU-time consumption effectively. But, <u>the accuracy of solution is affected much</u>.

↑

Higher Order Born Approximation

Higher Order Born Approximation

The simultaneous linear equations, $Ac = -g,$ can be solved step by step.

$$
\begin{aligned}
&A^{(0)}c^{(0)} = -g, \\
&A^{(0)}dc^{(1)} = -dAc^{(0)}, \\
&A^{(0)}dc^{(2)} = -dAdc^{(1)}, \\
&\cdots \\
&A^{(0)}dc^{(n)} = -dAdc^{(n-1)}, \\
&\cdots.
\end{aligned}
$$

Order

where $A^{(0)}$ is the principal part of A and dA is the rest $A - A^{(0)}$. The sum of both members,

$$\left(A^{(0)} + dA\right)\left(c^{(0)} + dc^{(1)} + dc^{(2)} + \cdots\right) = -g.$$

gives the final solution in the recursive way.

$$c = c^{(0)} + dc^{(1)} + dc^{(2)} + \cdots.$$

Order

Igel et. al (2000), Phys. Earth Planet. Inter., 119, 3-23

Idea:
Green's Function Matrix can be spread in two parts,
one is principal part that corresponds to the contribution of boundary elements nearby each other,

$$A = A^{(0)} + dA$$

and another is that of far boundary elements.

A way to improve the accuracy of the solution obtained by the sparse matrix approximation.

Numerical Example: Semi-spherical 3D valley

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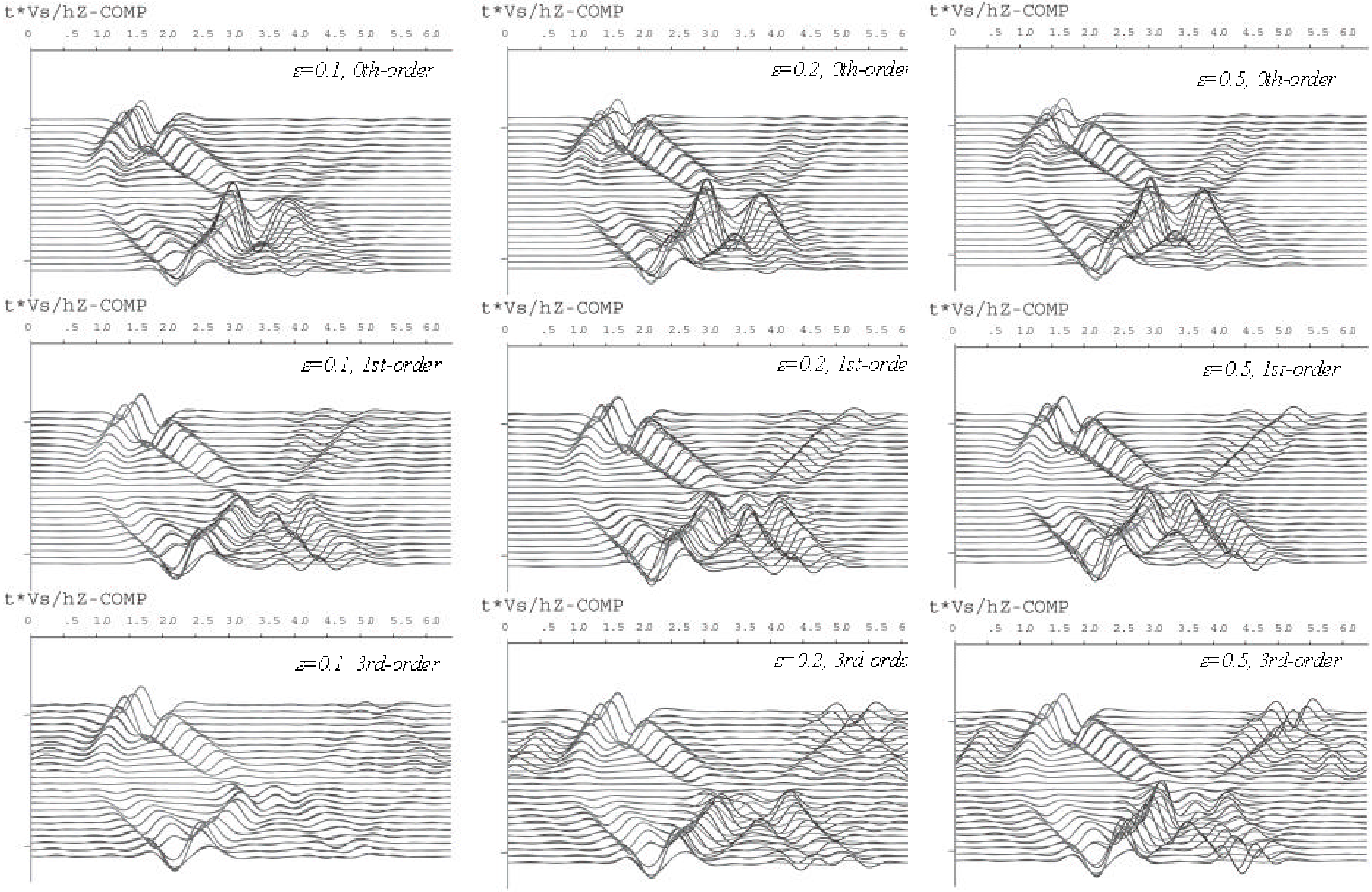
t*Vs/hZ-COMP
0 .5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0 5.5 6.0
ε=0.1, 0th-order
ε=0.2, 0th-order
ε=0.5, 0th-order
ε=0.1, 1st-order
ε=0.2, 1st-order
ε=0.5, 1st-order
ε=0.1, 3rd-order
ε=0.2, 3rd-order
ε=0.5, 3rd-order

Parameter that gives the truncation threshold

Table 1 Threshold level and corresponding distance from source element

e	0.1	0.2	0.5
Distance/Δr	4.1	2.0	0.8

Table 2 Performance of calculation

e	0.0	0.1			0.2			0.5		
Storage(%)	100	6.1			1.6			0.4		
Order	0	0	1	3	0	1	3	0	1	3
CPU time(%)	100	0.6	2.7	6.8	0.4	2.6	6.8	0.3	2.6	6.8

These are extreme examples!

$$A = A^{(0)} + dA$$

Contribution of Near Elements

+ Very Sparse Matrix
+ Easy to Calculate
+ Easy to solve simultaneous linear equations

$$A^{(0)}c^{(0)} = -g,$$

$$A^{(0)}dc^{(i)} = -dAc^{(i)},$$

Contribution of Far Elements

+ Used only for the product

$$dAc^{(i)}$$

+ Almost Full Matrix

  too big to be kept in the main memory

  too much CPU time consumed to calculate for each iteration

Possible measures:

to be kept in the main memory;

A compact way for saving?
Green’s function data base?

to be calculated for each iteration;

Fast Multi Pole Method

The way for the practical use is still far,

but we keep on being ambitious.